

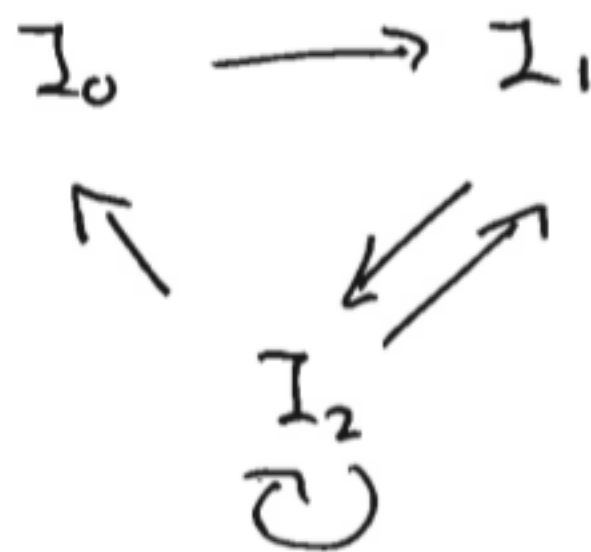
2. Let $F: I \rightarrow R$ have a 4-cycle $\{x_0, x_1, x_2, x_3\}$.

i) a) Suppose $x_0 < x_1 < x_2 < x_3$.

Let $I_0 = [x_0, x_1]$, $I_1 = [x_1, x_2]$, $I_2 = [x_2, x_3]$.

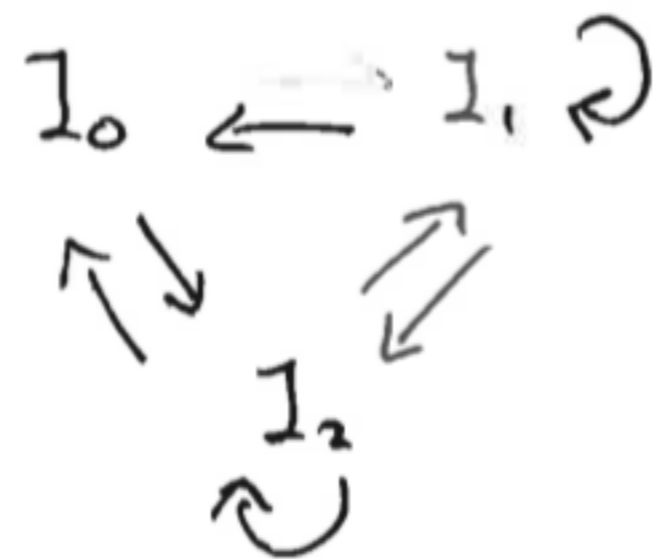
Then $F(I_0) \supseteq [F(x_0), F(x_1)] = [x_1, x_2] = I_1$
 $F(I_1) \supseteq [F(x_1), F(x_2)] = [x_2, x_3] = I_2$
 $F(I_2) \supseteq [F(x_3), F(x_2)] = [x_0, x_3] = I_0 \cup I_1 \cup I_2$.

Hence the transition graph Γ of f -covering relations is



This transition graph does not indicate a horseshoe.

However, the induced graph of f^2 -covering relations is



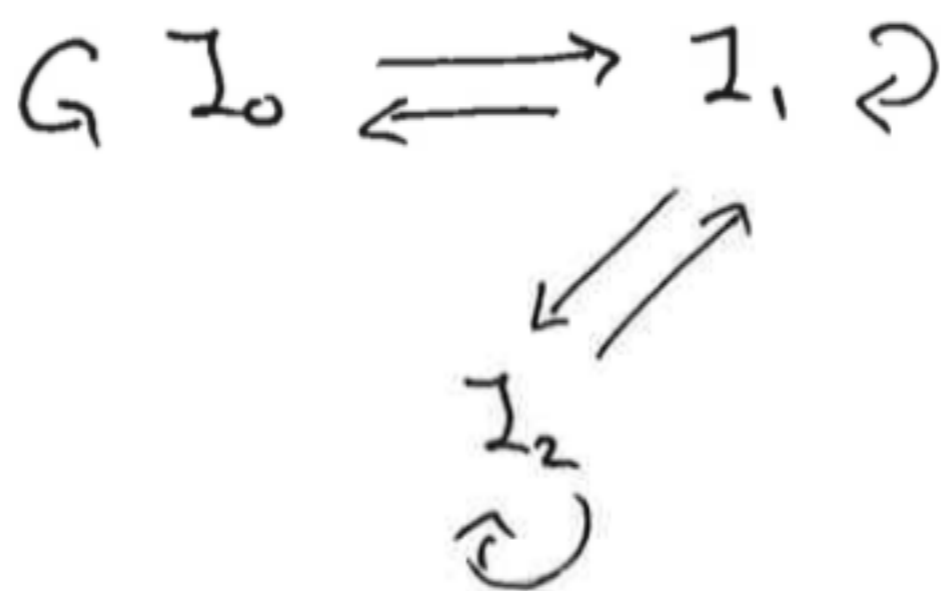
This graph has a subgraph (the I_1, I_2 section) ~~with~~ that indicates f^2 has a horseshoe.

Hence f is chaotic.

b) Suppose $x_1 < x_0 < x_2 < x_3$
 Let $I_0 = [x_1, x_0]$, $I_1 = [x_0, x_2]$, $I_2 = [x_2, x_3]$.

Then $f(I_0) \supseteq [x_1, x_2] = I_0 \cup I_1$
 $f(I_1) \supseteq [x_1, x_3] = I_0 \cup I_1 \cup I_2$
 $f(I_2) \supseteq [x_0, x_3] = I_1 \cup I_2$

Hence the transition graph is

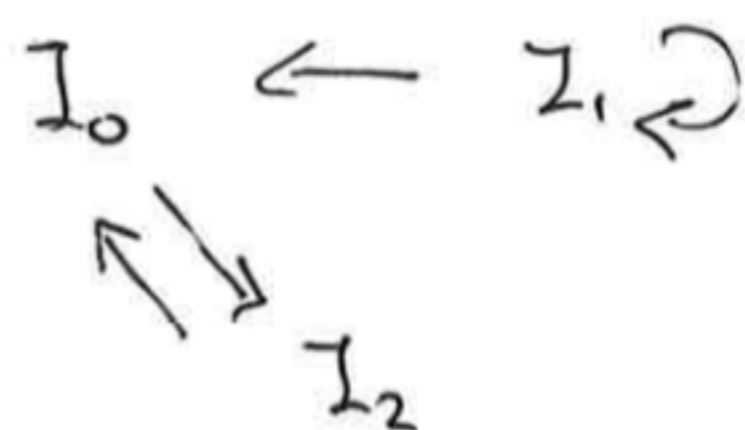


This graph indicates that f has two horseshoes, and so is chaotic.

c) Suppose $x_3 < x_1 < x_2 < x_0$
 Let $I_0 = [x_3, x_1]$, $I_1 = [x_1, x_2]$, $I_2 = [x_2, x_0]$

Then $f(I_0) \supseteq [x_2, x_0] = I_2$
 $f(I_1) \supseteq [x_3, x_2] = I_0 \cup I_1$
 $f(I_2) \supseteq [x_3, x_1] = I_0$

Hence the transition graph is



This graph does not show a horseshoe.
 Furthermore, $f^n(I_0)$ cannot cover for any n , so $\{I_0, I_1\}$ cannot form a horseshoe for any n .
 Furthermore, for any n , either $f^n(I_0) \rightarrow I_2$ and $f^n(I_2) \rightarrow I_0$ or $f^n(I_0) \rightarrow I_0$ and $f^n(I_2) \rightarrow I_2$, but not both.
 So $\{I_0, I_2\}$ cannot form a horseshoe for any n .
 Clearly $\{I_1, I_2\}$ cannot form a horseshoe for any n either.

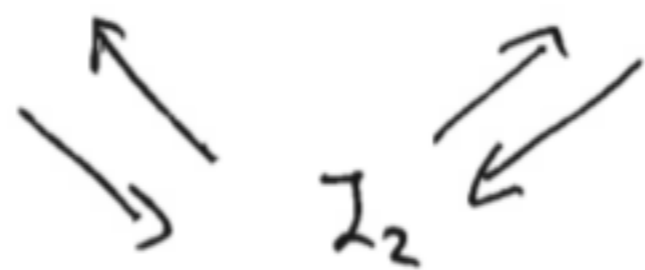
Hence F is not chaotic.

d). Suppose $x_1 < x_2 < x_0 < x_3$
 Let $I_0 = [x_1, x_2]$, $I_1 = [x_2, x_0]$, $I_2 = [x_0, x_3]$

Then $f(I_0) \supseteq I_1 \cup I_2$
 $f(I_1) \supseteq I_0 \cup I_2 \cup I_3$
 $f(I_2) \supseteq I_0 \cup I_1$

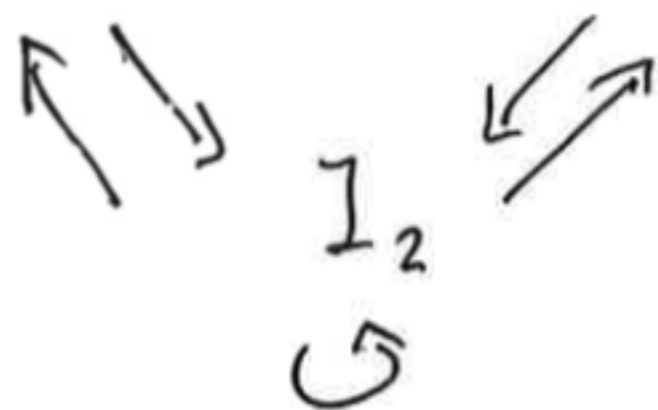
Hence the transition graph is

$$I_0 \rightleftharpoons I_1 \cup \emptyset$$



This graph does not show a horseshoe. However, the induced graph for f^2 is

$$G \rightarrow I_0 \rightleftharpoons I_1 \cup \emptyset$$



which has horseshoes everywhere. Hence F is chaotic.