4.

$$F: \mathbb{R}_+ \to \mathbb{R}_+, \qquad f(x) = 2x$$
$$G: \mathbb{R} \to \mathbb{R}, \qquad G(y) = y + \log(2).$$

<u>F and G conjugate</u>

Let $h(x) = \log(x)$.

Then
$$\begin{aligned} h \circ f(x) &= \log(2x) \\ &= \log(2) + \log(x) \\ &= \log(2) + h(x) = G(h(x)) \\ &= G \circ h(x). \end{aligned}$$

Hence f and G are conjugate via h (which is continuous on $\mathbb{R}^+$ and has a continuous inverse on $\mathbb{R}$).

SDIC

F has SDIC:

fix $\delta = 1$, say. for any $x > 0$ and $\varepsilon > 0$ choose $y > 0$ and $n \geq 1$ such that

$$\frac{\varepsilon}{2} < |x-y| < \varepsilon$$

and

$$\frac{1}{2^{n-1}} < \varepsilon.$$

Then $$|F(x)-F(y)| = 2|x-y|$$

$$\Rightarrow \quad |F^n(x)-F^n(y)| > 2^n \frac{\varepsilon}{2} > 1 = \delta.$$

But $$|x-y| < \varepsilon.$$

Hence F has SDIC.

G does not have SDIC:

for any $x, y \in \mathbb{R}$, $G(x) - G(y) = x - y$
$\Rightarrow G^n(x) - G^n(y) = x - y \quad \forall n \geq 1.$

Hence G does not have SDIC.

Hence SDIC is not preserved under Conjugacy.