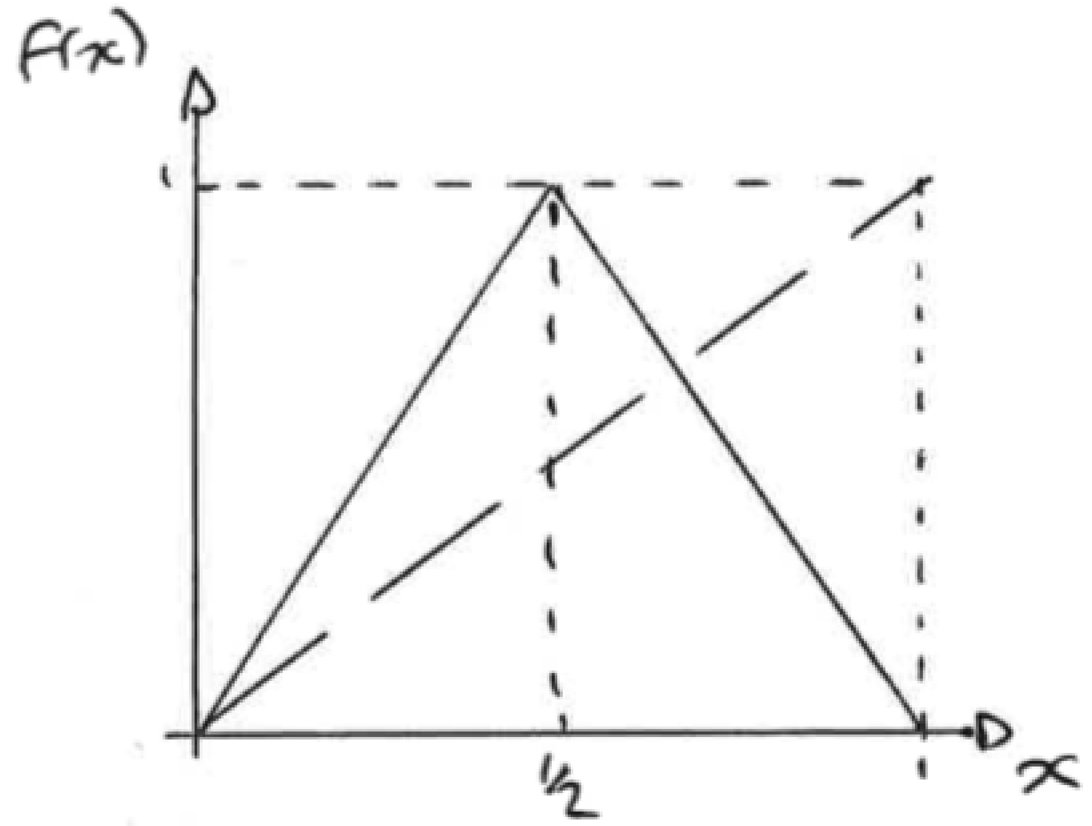
1. Tent map

$$x_{n+1} = f(x_n)$$

where
$$f(x) = \begin{cases} 2x, & 0 \le x \le \tfrac{1}{2} \\ 2(1-x), & \tfrac{1}{2} \le x \le 1 \end{cases}$$

i)



ii) Let the point $x \in [0,1]$ have binary expansion $x = 0.a_0 a_1 a_2 \ldots$

$0 \le x < \tfrac{1}{2}$ if and only if $a_0 = 0$.

Then $f(x) = 2x$, corresponds to shifting the binary expansion one place to the left: $f(x) = 0.a_1 a_2 a_3 \ldots$

$\tfrac{1}{2} \le x \le 1$ if and only if $a_1 = 0$.

Then $1-x$ corresponds to
$0.1111\ldots - 0.a_0 a_1 a_2 \ldots = 0.0\bar{a}_1 \bar{a}_2 \ldots$
where $\bar{a}_i = \begin{cases} 1 & \text{if } a_i = 0 \\ 0 & \text{if } a_i = 1. \end{cases}$

So $f(x) = 2(1-x)$ corresponds to
$f(x) = 0.\bar{a}_1 \bar{a}_2 \bar{a}_3 \ldots$

Hence the action of F is equivalent to the action of $\hat{\sigma}$ where

$$\hat{\sigma}(0.a_0a_1\ldots) = \begin{cases} 0.a_1a_2\ldots & , a_0=0 \\ 0.\bar{a}_1\bar{a}_2\ldots & , a_0=1 \end{cases}$$

and $\bar{a}_i = 1-a_i$.

iii) Points in 3-cycles under f correspond to symbol sequences that have period 3 under $\hat{\sigma}$.

Any such symbol sequence must contain at least one 0, and at least one 1. Hence it must contain the block 01.

Without loss of generality put this block at the start of the sequence

So, let $\quad x_0 = 0.01a_2a_3a_4a_5\ldots$

Then $\quad x_1 = 0.1a_2a_3a_4\ldots$

$\quad\quad x_2 = 0.\bar{a}_2\bar{a}_3\bar{a}_4\ldots$

There are two possibilities for x_3.
If $\bar{a}_2=0$, $a_2=1$ and

$$x_3 = 0.\bar{a}_3\bar{a}_4\bar{a}_5\ldots$$

Hence $\quad x_3=x_0$ if
$$\begin{aligned} \bar{a}_3 &= a_0 \Rightarrow a_3=1 \\ \bar{a}_4 &= a_1 \Rightarrow a_4=0 \\ \bar{a}_5 &= a_2 \Rightarrow a_5=0 \\ \bar{a}_6 &= a_3 \Rightarrow a_6=0 \quad \text{etc} \end{aligned}$$

Hence $\quad x_0 = 0.0111000111\ldots = \frac{64}{63}\left(\frac{7}{16}\right)=\frac{4}{9}.$

if $\bar{a}_2 = 1$, $a_2 = 0$ and

$$x_3 = 0 \cdot a_3 a_4 a_5 \ldots$$

Hence $x_3 = x_0$ if

$$\begin{aligned} a_3 &= a_0 = 0 \\ a_4 &= a_1 = 1 \\ a_5 &= a_2 = 0 \quad \text{etc} \end{aligned}$$

Hence $x_0 = 0 \cdot 0100010 \ldots = \frac{8}{7} \cdot \frac{2}{8} = \frac{2}{7}$.

The complete 3-cycles are:

$$\frac{4}{9} \to \frac{8}{9} \to \frac{2}{9} \to \frac{4}{9} \ldots$$

$$\frac{2}{7} \to \frac{4}{7} \to \frac{6}{7} \to \frac{2}{7} \ldots$$